Measurement of magneto-optical properties of materials with a Kerr spectrometer

M.S. Creative Component
Iowa State University
Department of Physics & Astro.
Kurt Jensen July 1995

Dielectric & conductivity tensors

For an otherwise optically isotropic magneto-optic material:

$$\overleftrightarrow{e} = \begin{pmatrix} \tilde{e} & -i\tilde{Q}\tilde{e} & 0 \\ i\tilde{Q}\tilde{e} & \tilde{e} & 0 \\ 0 & 0 & \tilde{e} \end{pmatrix} \quad \overleftrightarrow{s} = \begin{pmatrix} \tilde{s}_{xx} & -\tilde{s}_{xy} & 0 \\ \tilde{s}_{xy} & \tilde{s}_{xx} & 0 \\ 0 & 0 & \tilde{s}_{xx} \end{pmatrix}$$

Q is the magneto-optic Voigt parameter.
These tensors are related:

$$\overleftrightarrow{e} = e_0 + i\frac{4p\overleftrightarrow{s}}{w}$$

Complex Kerr angle

$$\tilde{f}_K \equiv q_K + ie_K = \frac{\tilde{E}_y}{\tilde{E}_x} = -\frac{i\tilde{n}\tilde{Q}}{1-\tilde{n}^2} = -\frac{\tilde{e}_{xy}}{(1-\tilde{e})\sqrt{\tilde{e}}}$$

$$= -\frac{4pi}{w}\frac{\tilde{s}_{xy}}{\tilde{n}(1-\tilde{n}^2)}$$

In a circular basis, this can be expressed as:

$$\tilde{f}_K = i\frac{\tilde{n}_+ - \tilde{n}_-}{\tilde{n}_+\tilde{n}_- - 1} = i\frac{\tilde{r}_+ - \tilde{r}_-}{\tilde{r}_+ + \tilde{r}_-}$$

Via Fresnel Eqn.

Complex reflectivity

The complex Fresnel reflectivity at normal incidence is:

$$\tilde{r}_{\pm} = \tilde{r}_x \pm i\tilde{r}_y = \frac{1-\tilde{n}_{\pm}}{1+\tilde{n}_{\pm}}$$

where the subscripts denote helicity in the usual sense.
The Kerr parameters in terms of the reflectivity are:

$$q_K = \frac{\Delta_- - \Delta_+}{2} \quad ; \quad e_K = \frac{r_- - r_+}{r_- + r_+}$$

where

$$\tilde{r}_{\pm} \equiv r_{\pm} e^{i\Delta_{\pm}}$$

The magnitude of the Kerr effect is proportional to magnetic field

$$n_- - n_+ = n(w + w_L) - n(w - w_L) = 2\frac{dn}{dw} w_L$$

$$w_L = \frac{eH}{2mc}$$ is the Larmor frequency

so $$f_K = i\frac{\tilde{n}_+ - \tilde{n}_-}{\tilde{n}_+ \tilde{n}_- - 1} \propto H$$

Instrument model

The Jones vector for the detected light is modeled as follows:

$$T = ASMP$$

$$P = \begin{pmatrix} \cos y \\ \sin y \end{pmatrix}$$

P is the Jones vector for the light transmitted by the polarizer. y is the polarizer angle.

$$M = \begin{pmatrix} 1 & 0 \\ 0 & e^{id} \end{pmatrix}$$

M is the Jones matrix for the modulator. d is the modulator retardation.

$$S = \begin{pmatrix} 1 & 1 \\ -i & i \end{pmatrix} \begin{pmatrix} r_+ e^{i\Delta_+} & 0 \\ 0 & r_- e^{i\Delta_-} \end{pmatrix} \begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix}$$

S is the sample reflectivity.

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \cos j & \sin j \\ -\sin j & \cos j \end{pmatrix}$$

A is the Jones matrix for the analyzer. φ is the analyzer angle.

Instrument model

The transmitted intensity is the square magnitude of *T*:

$$I = T^{*}T$$

We modulate the birefringence in M as $d=d_0 \sin\omega t$. Multiplying the matrices out yields the following for components of *I* at DC, *w*, and *2w*.

$$I_{DC} = 2R[1+\cos(2y)\cos(2j+2q_K) + J_0(d_0)\sin(2j+2q_K)]$$

$$I_w = -2\Delta R\, J_1(d_0) \qquad I_{2w} = 4R\, J_2(d_0)$$

Where $2R = r_+^2 + r_-^2 \,;\, \Delta R = r_+^2 - r_-^2$

$$r_+ r_- \approx R \,;\, q_K = \frac{\Delta_- - \Delta_+}{2}$$

Error propagation

The Kerr signal is proportional to the AC component normalized to the DC level:

$$f = C\frac{V_{AC}}{V_{DC}} \quad ; \quad V_{DC} = V_L - V_D$$

We measure the variance of each measured voltage to find the total variance in the Kerr signal.

$$(df)^2 = C^2\left(\frac{\P f}{\P V_{AC}}\right)^2 (dV_{AC})^2 + \left(\frac{\P f}{\P V_L}\right)^2 (dV_L)^2 + \left(\frac{\P f}{\P V_D}\right)^2 (dV_D)^2$$

This gives

$$(df)^2 = C^2\frac{(dV_{AC})^2}{V_{DC}^2} + \frac{V_{AC}^2}{V_{DC}^4}\left[(dV_L)^2 + (dV_D)^2\right]$$